

1.2 Solving Multi-Step Equations



Learning Standards
HSN-Q.A.1
HSA-CED.A.1
HSA-REI.B.3

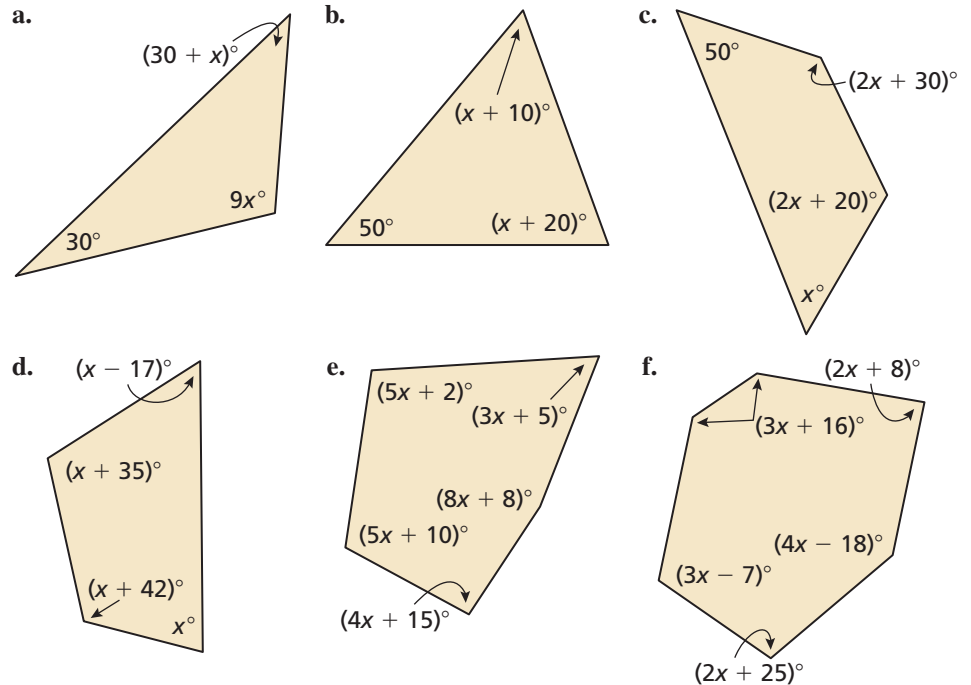
JUSTIFYING CONCLUSIONS

To be proficient in math, you need to be sure your answers make sense in the context of the problem. For instance, if you find the angle measures of a triangle, and they have a sum that is not equal to 180° , then you should check your work for mistakes.

Essential Question How can you use multi-step equations to solve real-life problems?

EXPLORATION 1 Solving for the Angle Measures of a Polygon

Work with a partner. The sum S of the angle measures of a polygon with n sides can be found using the formula $S = 180(n - 2)$. Write and solve an equation to find each value of x . Justify the steps in your solution. Then find the angle measures of each polygon. How can you check the reasonableness of your answers?



EXPLORATION 2 Writing a Multi-Step Equation

Work with a partner.

- Draw an irregular polygon.
- Measure the angles of the polygon. Record the measurements on a separate sheet of paper.
- Choose a value for x . Then, using this value, work backward to assign a variable expression to each angle measure, as in Exploration 1.
- Trade polygons with your partner.
- Solve an equation to find the angle measures of the polygon your partner drew. Do your answers seem reasonable? Explain.

Communicate Your Answer

- How can you use multi-step equations to solve real-life problems?
- In Exploration 1, you were given the formula for the sum S of the angle measures of a polygon with n sides. Explain why this formula works.
- The sum of the angle measures of a polygon is 1080° . How many sides does the polygon have? Explain how you found your answer.

1.2 Lesson

What You Will Learn

- ▶ Solve multi-step linear equations using inverse operations.
- ▶ Use multi-step linear equations to solve real-life problems.
- ▶ Use unit analysis to model real-life problems.

Core Vocabulary

Previous
inverse operations
mean

Solving Multi-Step Linear Equations

Core Concept

Solving Multi-Step Equations

To solve a multi-step equation, simplify each side of the equation, if necessary. Then use inverse operations to isolate the variable.

EXAMPLE 1 Solving a Two-Step Equation

Solve $2.5x - 13 = 2$. Check your solution.

SOLUTION

$$2.5x - 13 = 2$$

Write the equation.

Undo the subtraction.

$$+ 13 \quad + 13$$

Add 13 to each side.

$$2.5x = 15$$

Simplify.

Undo the multiplication.

$$\frac{2.5x}{2.5} = \frac{15}{2.5}$$

Divide each side by 2.5.

$$x = 6$$

Simplify.

Check

$$2.5x - 13 = 2$$

$$2.5(6) - 13 \stackrel{?}{=} 2$$

$$2 = 2 \quad \checkmark$$

- ▶ The solution is $x = 6$.

EXAMPLE 2 Combining Like Terms to Solve an Equation

Solve $-12 = 9x - 6x + 15$. Check your solution.

SOLUTION

$$-12 = 9x - 6x + 15$$

Write the equation.

$$-12 = 3x + 15$$

Combine like terms.

Undo the addition.

$$-15 \quad -15$$

Subtract 15 from each side.

$$-27 = 3x$$

Simplify.

Undo the multiplication.

$$\frac{-27}{3} = \frac{3x}{3}$$

Divide each side by 3.

$$-9 = x$$

Simplify.

Check

$$-12 = 9x - 6x + 15$$

$$-12 \stackrel{?}{=} 9(-9) - 6(-9) + 15$$

$$-12 = -12 \quad \checkmark$$

- ▶ The solution is $x = -9$.

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Solve the equation. Check your solution.

1. $-2n + 3 = 9$

2. $-21 = \frac{1}{2}c - 11$

3. $-2x - 10x + 12 = 18$

EXAMPLE 3 Using Structure to Solve a Multi-Step EquationSolve $2(1 - x) + 3 = -8$. Check your solution.**SOLUTION****Method 1** One way to solve the equation is by using the Distributive Property.

$$\begin{array}{ll}
 2(1 - x) + 3 = -8 & \text{Write the equation.} \\
 2(1) - 2(x) + 3 = -8 & \text{Distributive Property} \\
 2 - 2x + 3 = -8 & \text{Multiply.} \\
 -2x + 5 = -8 & \text{Combine like terms.} \\
 \underline{-5} \quad \underline{-5} & \text{Subtract 5 from each side.} \\
 -2x = -13 & \text{Simplify.} \\
 \frac{-2x}{-2} = \frac{-13}{-2} & \text{Divide each side by } -2. \\
 x = 6.5 & \text{Simplify.}
 \end{array}$$

▶ The solution is $x = 6.5$.**Check**

$$\begin{array}{l}
 2(1 - x) + 3 = -8 \\
 2(1 - 6.5) + 3 \stackrel{?}{=} -8 \\
 -8 = -8 \quad \checkmark
 \end{array}$$

Method 2 Another way to solve the equation is by interpreting the expression $1 - x$ as a single quantity.

$$\begin{array}{ll}
 2(1 - x) + 3 = -8 & \text{Write the equation.} \\
 \underline{-3} \quad \underline{-3} & \text{Subtract 3 from each side.} \\
 2(1 - x) = -11 & \text{Simplify.} \\
 \frac{2(1 - x)}{2} = \frac{-11}{2} & \text{Divide each side by 2.} \\
 1 - x = -5.5 & \text{Simplify.} \\
 \underline{-1} \quad \underline{-1} & \text{Subtract 1 from each side.} \\
 -x = -6.5 & \text{Simplify.} \\
 \frac{-x}{-1} = \frac{-6.5}{-1} & \text{Divide each side by } -1. \\
 x = 6.5 & \text{Simplify.}
 \end{array}$$

▶ The solution is $x = 6.5$, which is the same solution obtained in Method 1.**LOOKING FOR STRUCTURE**First solve for the expression $1 - x$, and then solve for x .**Monitoring Progress**  Help in English and Spanish at BigIdeasMath.com

Solve the equation. Check your solution.

4. $3(x + 1) + 6 = -9$
5. $15 = 5 + 4(2d - 3)$
6. $13 = -2(y - 4) + 3y$
7. $2x(5 - 3) - 3x = 5$
8. $-4(2m + 5) - 3m = 35$
9. $5(3 - x) + 2(3 - x) = 14$

Solving Real-Life Problems

EXAMPLE 4 Modeling with Mathematics



Use the table to find the number of miles x you need to bike on Friday so that the mean number of miles biked per day is 5.

Day	Miles
Monday	3.5
Tuesday	5.5
Wednesday	0
Thursday	5
Friday	x

SOLUTION

- 1. Understand the Problem** You know how many miles you biked Monday through Thursday. You are asked to find the number of miles you need to bike on Friday so that the mean number of miles biked per day is 5.
- 2. Make a Plan** Use the definition of mean to write an equation that represents the problem. Then solve the equation.
- 3. Solve the Problem** The mean of a data set is the sum of the data divided by the number of data values.

$$\frac{3.5 + 5.5 + 0 + 5 + x}{5} = 5$$

Write the equation.

$$\frac{14 + x}{5} = 5$$

Combine like terms.

$$5 \cdot \frac{14 + x}{5} = 5 \cdot 5$$

Multiply each side by 5.

$$14 + x = 25$$

Simplify.

$$\underline{-14} \quad \underline{-14}$$

Subtract 14 from each side.

$$x = 11$$

Simplify.

► You need to bike 11 miles on Friday.

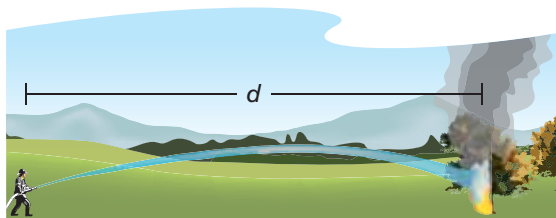
- 4. Look Back** Notice that on the days that you did bike, the values are close to the mean. Because you did not bike on Wednesday, you need to bike about twice the mean on Friday. Eleven miles is about twice the mean. So, your solution is reasonable.

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- 10.** The formula $d = \frac{1}{2}n + 26$ relates the nozzle pressure n (in pounds per square inch) of a fire hose and the maximum horizontal distance the water reaches d (in feet). How much pressure is needed to reach a fire 50 feet away?



REMEMBER

When you add miles to miles, you get miles. But, when you divide miles by days, you get miles per day.



Using Unit Analysis to Model Real-Life Problems

When you write an equation to model a real-life problem, you should check that the units on each side of the equation balance. For instance, in Example 4, notice how the units balance.

$$\frac{3.5 + 5.5 + 0 + 5 + x}{5} = 5$$

Units: miles, miles per day, per, days

$$\frac{\text{mi}}{\text{day}} = \frac{\text{mi}}{\text{day}} \quad \checkmark$$

EXAMPLE 5 Solving a Real-Life Problem

Your school's drama club charges \$4 per person for admission to a play. The club borrowed \$400 to pay for costumes and props. After paying back the loan, the club has a profit of \$100. How many people attended the play?

SOLUTION

- Understand the Problem** You know how much the club charges for admission. You also know how much the club borrowed and its profit. You are asked to find how many people attended the play.
- Make a Plan** Use a verbal model to write an equation that represents the problem. Then solve the equation.
- Solve the Problem**

REMEMBER

When you multiply dollars per person by people, you get dollars.



Words Ticket price • Number of people who attended - Amount of loan = Profit

Variable Let x be the number of people who attended.

Equation $\frac{\$4}{\text{person}} \cdot x \text{ people} - \$400 = \$100$ \$ = \$ \checkmark

$$4x - 400 = 100 \quad \text{Write the equation.}$$

$$4x - 400 + 400 = 100 + 400 \quad \text{Add 400 to each side.}$$

$$4x = 500 \quad \text{Simplify.}$$

$$\frac{4x}{4} = \frac{500}{4} \quad \text{Divide each side by 4.}$$

$$x = 125 \quad \text{Simplify.}$$

▶ So, 125 people attended the play.

- Look Back** To check that your solution is reasonable, multiply \$4 per person by 125 people. The result is \$500. After paying back the \$400 loan, the club has \$100, which is the profit.



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- You have 96 feet of fencing to enclose a rectangular pen for your dog. To provide sufficient running space for your dog to exercise, the pen should be three times as long as it is wide. Find the dimensions of the pen.

1.2 Exercises

Vocabulary and Core Concept Check

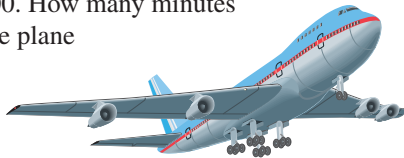
- COMPLETE THE SENTENCE** To solve the equation $2x + 3x = 20$, first combine $2x$ and $3x$ because they are _____.
- WRITING** Describe two ways to solve the equation $2(4x - 11) = 10$.

Monitoring Progress and Modeling with Mathematics

In Exercises 3–14, solve the equation. Check your solution. (See Examples 1 and 2.)

- $3w + 7 = 19$
- $2g - 13 = 3$
- $11 = 12 - q$
- $10 = 7 - m$
- $5 = \frac{z}{-4} - 3$
- $\frac{a}{3} + 4 = 6$
- $\frac{h + 6}{5} = 2$
- $\frac{d - 8}{-2} = 12$
- $8y + 3y = 44$
- $36 = 13n - 4n$
- $12v + 10v + 14 = 80$
- $6c - 8 - 2c = -16$

15. **MODELING WITH MATHEMATICS** The altitude a (in feet) of a plane t minutes after liftoff is given by $a = 3400t + 600$. How many minutes after liftoff is the plane at an altitude of 21,000 feet?



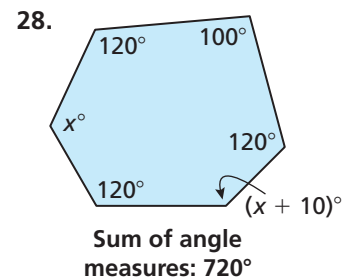
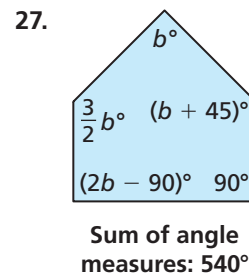
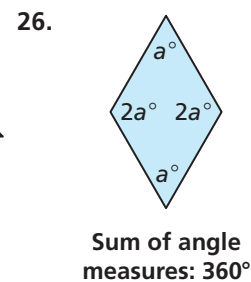
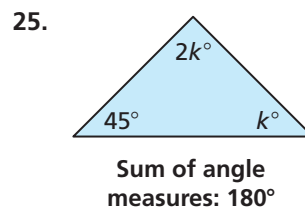
16. **MODELING WITH MATHEMATICS** A repair bill for your car is \$553. The parts cost \$265. The labor cost is \$48 per hour. Write and solve an equation to find the number of hours of labor spent repairing the car.

In Exercises 17–24, solve the equation. Check your solution. (See Example 3.)

- $4(z + 5) = 32$
- $-2(4g - 3) = 30$
- $6 + 5(m + 1) = 26$
- $5h + 2(11 - h) = -5$
- $27 = 3c - 3(6 - 2c)$
- $-3 = 12y - 5(2y - 7)$

- $-3(3 + x) + 4(x - 6) = -4$
- $5(r + 9) - 2(1 - r) = 1$

USING TOOLS In Exercises 25–28, find the value of the variable. Then find the angle measures of the polygon. Use a protractor to check the reasonableness of your answer.



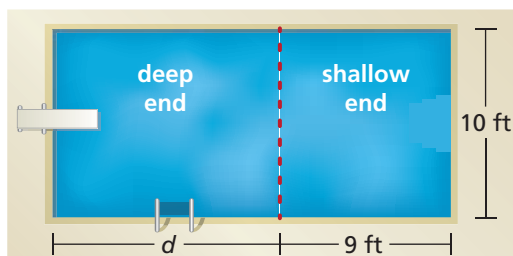
In Exercises 29–34, write and solve an equation to find the number.

- The sum of twice a number and 13 is 75.
- The difference of three times a number and 4 is -19 .
- Eight plus the quotient of a number and 3 is -2 .
- The sum of twice a number and half the number is 10.
- Six times the sum of a number and 15 is -42 .
- Four times the difference of a number and 7 is 12.

USING EQUATIONS In Exercises 35–37, write and solve an equation to answer the question. Check that the units on each side of the equation balance. (See Examples 4 and 5.)

35. During the summer, you work 30 hours per week at a gas station and earn \$8.75 per hour. You also work as a landscaper for \$11 per hour and can work as many hours as you want. You want to earn a total of \$400 per week. How many hours must you work as a landscaper?

36. The area of the surface of the swimming pool is 210 square feet. What is the length d of the deep end (in feet)?



37. You order two tacos and a salad. The salad costs \$2.50. You pay 8% sales tax and leave a \$3 tip. You pay a total of \$13.80. How much does one taco cost?

JUSTIFYING STEPS In Exercises 38 and 39, justify each step of the solution.

38. $-\frac{1}{2}(5x - 8) - 1 = 6$ Write the equation.

$-\frac{1}{2}(5x - 8) = 7$ _____

$5x - 8 = -14$ _____

$5x = -6$ _____

$x = -\frac{6}{5}$ _____

39. $2(x + 3) + x = -9$ Write the equation.

$2(x) + 2(3) + x = -9$ _____

$2x + 6 + x = -9$ _____

$3x + 6 = -9$ _____

$3x = -15$ _____

$x = -5$ _____

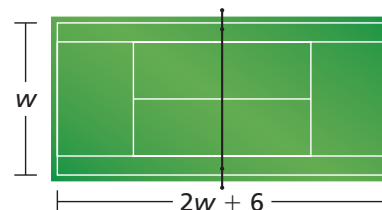
ERROR ANALYSIS In Exercises 40 and 41, describe and correct the error in solving the equation.

40.
$$\begin{aligned} -2(7 - y) + 4 &= -4 \\ -14 - 2y + 4 &= -4 \\ -10 - 2y &= -4 \\ -2y &= 6 \\ y &= -3 \end{aligned}$$

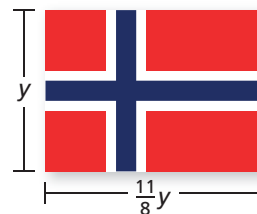
41.
$$\begin{aligned} \frac{1}{4}(x - 2) + 4 &= 12 \\ \frac{1}{4}(x - 2) &= 8 \\ x - 2 &= 2 \\ x &= 4 \end{aligned}$$

MATHEMATICAL CONNECTIONS In Exercises 42–44, write and solve an equation to answer the question.

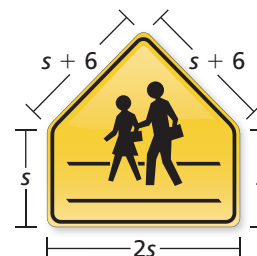
42. The perimeter of the tennis court is 228 feet. What are the dimensions of the court?



43. The perimeter of the Norwegian flag is 190 inches. What are the dimensions of the flag?



44. The perimeter of the school crossing sign is 102 inches. What is the length of each side?



45. **COMPARING METHODS** Solve the equation $2(4 - 8x) + 6 = -1$ using (a) Method 1 from Example 3 and (b) Method 2 from Example 3. Which method do you prefer? Explain.
46. **PROBLEM SOLVING** An online ticket agency charges the amounts shown for basketball tickets. The total cost for an order is \$220.70. How many tickets are purchased?

Charge	Amount
Ticket price	\$32.50 per ticket
Convenience charge	\$3.30 per ticket
Processing charge	\$5.90 per order

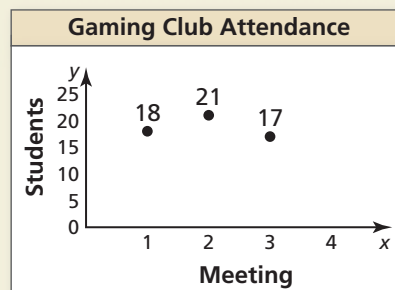
47. **MAKING AN ARGUMENT** You have quarters and dimes that total \$2.80. Your friend says it is possible that the number of quarters is 8 more than the number of dimes. Is your friend correct? Explain.

48. **THOUGHT PROVOKING** You teach a math class and assign a weight to each component of the class. You determine final grades by totaling the products of the weights and the component scores. Choose values for the remaining weights and find the necessary score on the final exam for a student to earn an A (90%) in the class, if possible. Explain your reasoning.

Component	Student's score	Weight	Score \times Weight
Class Participation	92%	0.20	$92\% \times 0.20 = 18.4\%$
Homework	95%		
Midterm Exam	88%		
Final Exam			
Total		1	

49. **REASONING** An even integer can be represented by the expression $2n$, where n is any integer. Find three consecutive even integers that have a sum of 54. Explain your reasoning.

50. **HOW DO YOU SEE IT?** The scatter plot shows the attendance for each meeting of a gaming club.



- The mean attendance for the first four meetings is 20. Is the number of students who attended the fourth meeting greater than or less than 20? Explain.
- Estimate the number of students who attended the fourth meeting.
- Describe a way you can check your estimate in part (b).

REASONING In Exercises 51–56, the letters a , b , and c represent nonzero constants. Solve the equation for x .

- $bx = -7$
- $x + a = \frac{3}{4}$
- $ax - b = 12.5$
- $ax + b = c$
- $2bx - bx = -8$
- $cx - 4b = 5b$

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Simplify the expression. (*Skills Review Handbook*)

57. $4m + 5 - 3m$

58. $9 - 8b + 6b$

59. $6t + 3(1 - 2t) - 5$

Determine whether (a) $x = -1$ or (b) $x = 2$ is a solution of the equation. (*Skills Review Handbook*)

60. $x - 8 = -9$

61. $x + 1.5 = 3.5$

62. $2x - 1 = 3$

63. $3x + 4 = 1$

64. $x + 4 = 3x$

65. $-2(x - 1) = 1 - 3x$